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Generalized Lyapunov Demodulator for Amplitude and Phase Estimation by the Internal Model Principle

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Outline

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Introduction

— Some applications require high-bandwidth demodulation.

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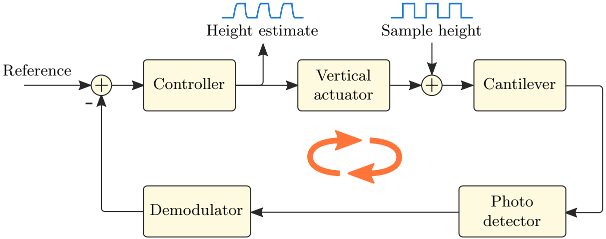

Atomic force microscopy (AFM) in dynamic mode.

Introduction

- The *Lyapunov demodulator* already achieves a very high demodulation bandwidth.
- However, compares unfavorably in terms of *off-mode rejection*.
  - Requirement in e.g. multi-frequency AFM.

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  - Requirement in e.g. multi-frequency AFM.

Here we propose a *generalized Lyapunov demodulator*, enabling direct filtering design.
⇒ Achieves increased off-mode rejection by employing higher-order filters.

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Standard Lyapunov demodulator

Demodulation problem

$$r(t) = a(t)\sin(\omega_c t + \varphi(t)) \tag{1}$$

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The standard Lyapunov demodulator can be written as

$$\begin{aligned} \dot{\boldsymbol{x}} &= \gamma \boldsymbol{c}\varepsilon, \\ y &= \boldsymbol{c}^T \boldsymbol{x}, \end{aligned} \quad (2)$$

where $\varepsilon = r - y$ and

$$\boldsymbol{c} = [\cos(\omega_c t),\ \sin(\omega_c t)]^T . \quad (3)$$

Amplitude and phase can be recovered from

$$\hat{a} = \sqrt{x_1^2 + x_2^2}, \quad \hat{\varphi} = \mathrm{atan2}\left(x_1, x_2\right). \quad (4)$$

Generalized Lyapunov demodulator

Sinusoidal signal $r(t)$ generated by the output of:

$$\begin{aligned} \dot{\boldsymbol{w}} &= \mathbf{S}\boldsymbol{w} \\ \boldsymbol{w}(0) &= \boldsymbol{w}_0 \\ r(t) &= \boldsymbol{\Gamma}^T \boldsymbol{w} \end{aligned} \tag{5}$$

with $\boldsymbol{\Gamma} = [1, 0]^T$ and

$$\mathbf{S} = \begin{bmatrix} 0 & \omega_c \\ -\omega_c & 0 \end{bmatrix}. \tag{6}$$

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Standard Lyapunov demodulator equivalently recast by the change of coordinates $\boldsymbol{v} = e^{\mathbf{S}t}\boldsymbol{x}$, which gives

$$\begin{aligned} \dot{\boldsymbol{v}} &= \mathbf{S}\boldsymbol{v} + \gamma\mathbf{\Gamma}\varepsilon \\ y &= \mathbf{\Gamma}^T \boldsymbol{v}. \end{aligned} \qquad (7)$$

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Replace ϵ by a filtered version, for additional design degrees-of-freedom ⇒

Generalized Lyapunov demodulator

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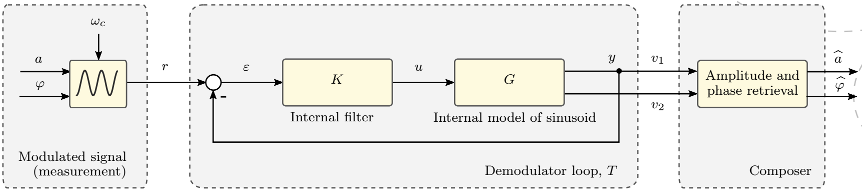
$$\begin{aligned} \dot{\boldsymbol{\eta}} &= \mathbf{A}\boldsymbol{\eta} + \mathbf{B}\varepsilon \\ \dot{\boldsymbol{v}} &= \mathbf{S}\boldsymbol{v} + \boldsymbol{\Gamma}\mathbf{C}\boldsymbol{\eta} \\ y &= \boldsymbol{\Gamma}^T\boldsymbol{v}. \end{aligned} \tag{8}$$

where **A**, **B**, **C** can be freely chosen to meet some design specifications.

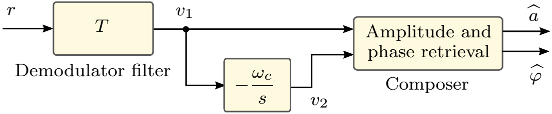
Indirect filter design



- Design $K(s)$ such that the demodulator loop $T(s)$ becomes a desired bandpass shape.
- Perfect tracking is guaranteed for any stable $K(s)$.

Direct filter design

Direct filter design



- Design $T(s)$ directly as a bandpass filter.
- Perfect tracking is guaranteed by the condition $T(j\omega_c) = 1$.

⇒ Approach taken in this work.

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Simulation Results

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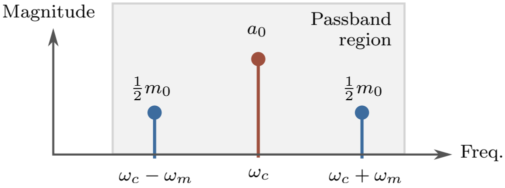
Filter design considerations



- Bandwidth.
- Relative filter order.
- Phase delay.
- Group delay.

Example filter implementations I

Higher-order Lyapunov demodulators

The standard Lyapunov demodulator represented in the generalized scheme:

$$T_1(s) = \frac{\gamma s}{s^2 + \gamma s + \omega_c^2}. \tag{9}$$

Example filter implementations I

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The higher order Lyapunov demodulators are then formulated as

$$T_i(s) = T_1(s)^i \tag{10}$$

where *i* represents the relative order of the filter.

Example filter implementations II

Bandpass form of the standard filters

- Butterworth filter
- Bessel filter
- Chebyshev type-I filter

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Simulation Procedure

Compare 3 kHz and 30 kHz bandwidth settings of:

- Relative order 1 Lyapunov filter (standard).
- Relative order 3 Lyapunov filter (higher-order).
- Relative order 3 Butterworth, Bessel, Chebyshev filters.

With carrier frequency 50 kHz.

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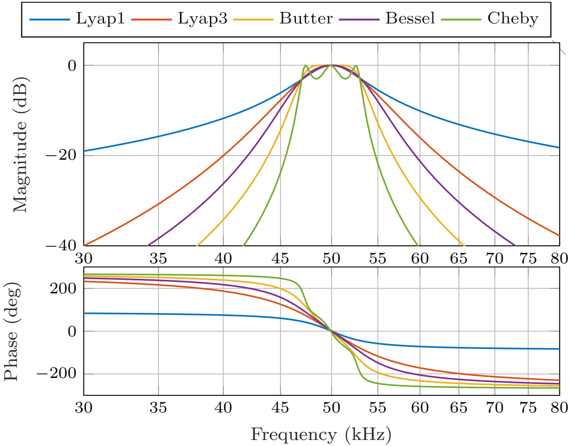
In terms of:

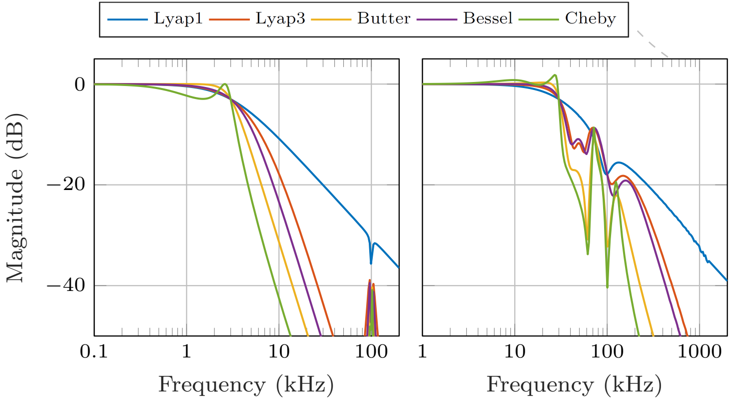
- Off-mode rejection.
- Transient tracking performance.

Bode plot $T(s)$



3 kHz bandwidth

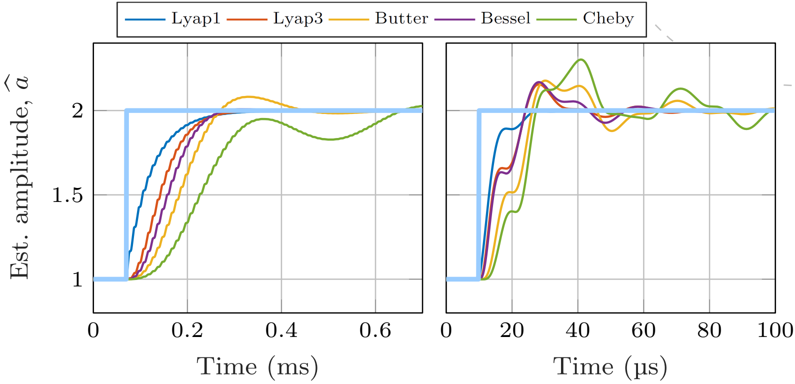
Tracking frequency response



3 kHz bandwidth

30 kHz bandwidth

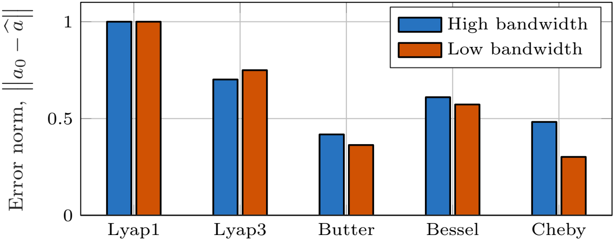
Step response



3 kHz bandwidth

30 kHz bandwidth

Off-mode rejection



Attenuation of harmonic frequency components outside the tracking bandwidth

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Questions?

Bibliography

Ragazzon, Michael R P et al. (2019). “Generalized Lyapunov Demodulator for Amplitude and Phase Estimation by the Internal Model Principle”. In *Proc. IFAC Mechatronics*. Vienna, Austria.