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$\mathcal{H}_\infty$ Reduced Order Control

Numerical Implementability

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Introduction

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- — Challenging high resonant peaks.
- — For good performance, we employ higher-order model-matching based techniques, such as $\mathcal{H}_\infty$ control.

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- Consider nanopositioning as an example application.
- Challenging high resonant peaks.
- For good performance, we employ higher-order model-matching based techniques, such as $\mathcal{H}_\infty$ control.
- Numerical solvers require sufficiently small step-size for stability.

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Challenge

Controllers of high order can be challenging to implement in real-time due to computational complexity.

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Outline

Robust Controller Design

We design a robustly stable $\mathcal{H}_\infty$ controller which is used later for model reduction.
Procedure:

— System identification

— $\mathcal{H}_\infty$ controller design

— Robust stability

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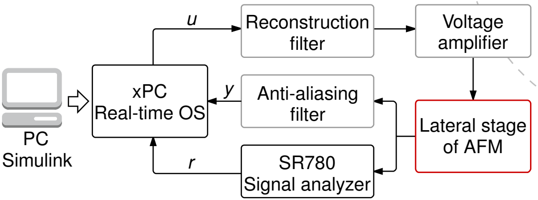
Experimental Setup



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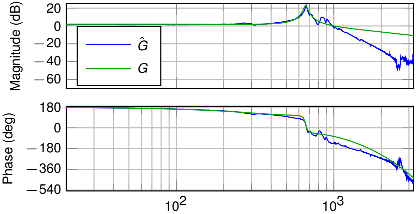
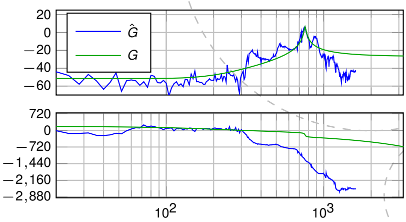
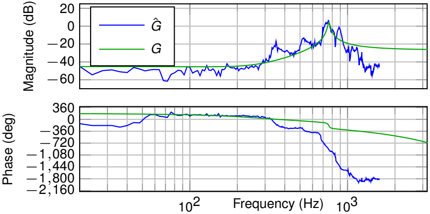
Frequency Response and Model Fit



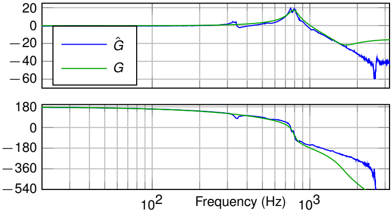
(a) G_{xx}



(b) G_{xy}



(c) G_{yx}



(d) G_{yy}

Controller Design

We have employed $\mathcal{H}_\infty$ mixed sensitivity control: Find K which satisfies

$$\min_K N(K) = \left\| \begin{matrix} W_1 S \\ W_2 T \\ W_3 KS \end{matrix} \right\|_\infty \qquad (1)$$

where

— W_1, W_2, and W_3 are user-defined frequency-dependent weightings.

— S, T is the closed-loop sensitivity and complementary sensitivity function respectively.

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Resulting controller properties after specifying weights and solving (1).

	ω_{BS} [Hz]	ω_{BT} [Hz]	$\|WT\|_\infty$	Model order
PID	58.0	93.1	1.073	4th-order
$\mathcal{H}_\infty$	69.8	98.6	0.6717	18th-order

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Outline

Control Order Reduction

— Removing states from the controller model with minimal impact on the qualitative response.

— Purpose: To reduce the computational complexity of the controller.

— A common method is balanced residualization.

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Control Order Reduction

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Next, we apply this method to our $\mathcal{H}_\infty$ controller to various reduced orders.

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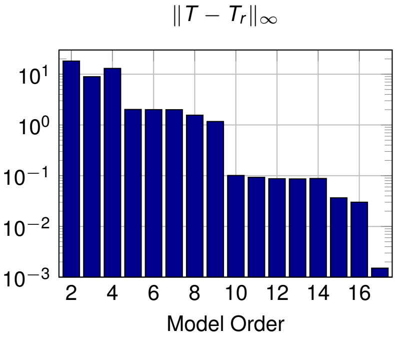
Control Order Reduction Results



(a) Closed-loop error.
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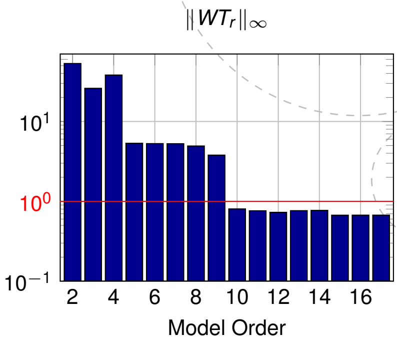

(b) Robustness.
Must be < 1 for robust stability.

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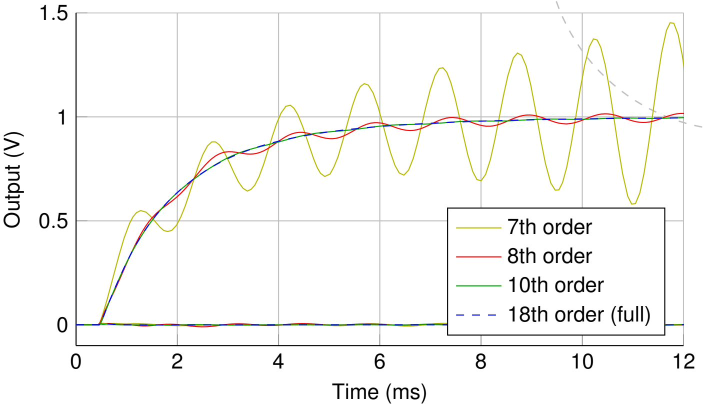
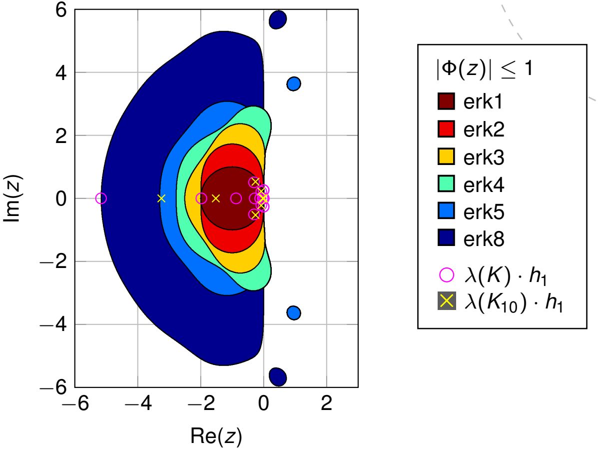
Control Order Reduction Results



Simulated step-response

Control Order Reduction Results



Where $\Phi(z)$ is the stability function of the given ERK method,

$$\dot{y} = \lambda y \quad \Rightarrow \quad y_{n+1} = \Phi(h\lambda) y_n.$$

Control Order Reduction Results

Model Reduction and Numerical Stability

Not only does model reduction reduce the computational complexity of the controller, it may also allow for a larger step-size.

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Maximum Step-Size

Maximum step-size for a given explicit Runge-Kutta (ERK) method.

Controller Order	h_{max} [µs]					
	erk1	erk2	erk3	erk4	erk5	erk8
7 (unst.)	32.18	71.33	95.74	100.8	127.2	203.6
8	58.37	58.37	73.34	81.29	96.51	150.8
9	37.54	37.54	47.17	52.28	62.07	96.99
10	33.61	33.61	42.22	46.8	55.56	86.82
11	39.70	39.7	49.87	55.28	65.63	102.5
12	24.96	32.66	41.03	45.48	54.00	84.37
13	2.821	21.55	27.07	30.01	35.63	55.67
14	2.818	19.11	24.01	26.62	31.60	49.38
15	2.843	21.73	27.30	30.26	35.92	56.13
16	2.948	22.64	28.44	31.53	37.43	58.48
17	2.918	21.18	26.61	29.49	35.01	54.71
18 (full)	2.910	21.14	26.56	29.45	34.96	54.62
PID	80.00	80.00	100.5	111.4	132.3	206.7

Outline

Experimental Results

Experiments were performed for two reasons:

1. To compare the reduced order controllers in terms of performance.
2. To record the average task execution time (TET).
   - A measurement of the computational complexity of the controller.
   - A lower limit on the chosen solver step-size.

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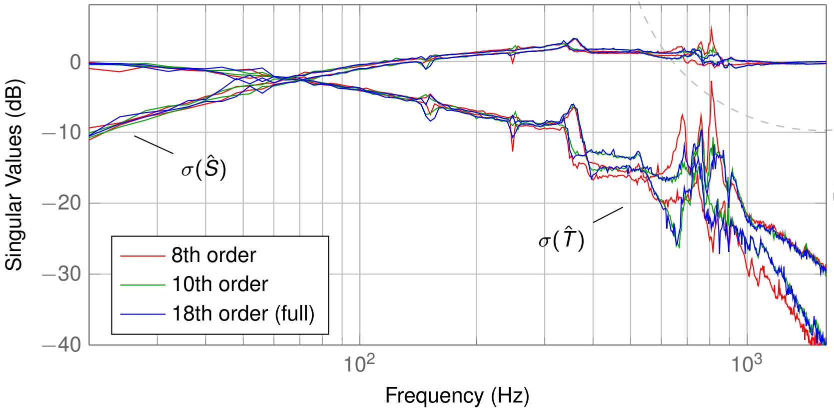
Closed-Loop Performance



Singular values of closed-loop frequency response, $\sigma(\hat{S})$ and $\sigma(\hat{T})$.

Computational Complexity

Average task execution time (TET). Step-size $h = 40\,\mu s$. Dash (-) unstable, not tested.

Controller Order	Average TET [µs]				
	erk1	erk2	erk3	erk5	erk8
≤ 7	-	-	-	-	-
8	-	10.25	10.43	11.10	13.64
9	-	-	10.55	11.43	14.26
10	-	-	10.71	11.64	14.99
11	-	-	10.89	11.91	15.79
12	-	-	11.07	12.36	16.79
13	-	-	-	-	17.68
14	-	-	-	-	18.82
15	-	-	-	-	19.61
16	-	-	-	-	20.91
17	-	-	-	-	22.61
18	-	-	-	-	20.11
PID	-	9.95	9.98	10.13	11.12

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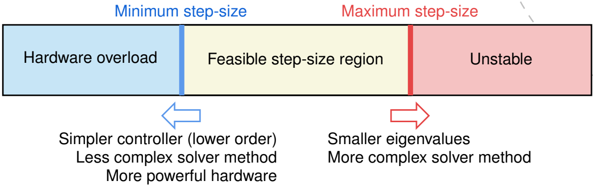
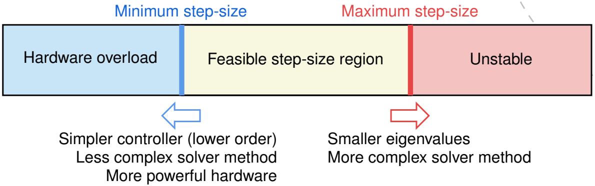
Conclusion


Minimum step-size
Maximum step-size
Hardware overload
Feasible step-size region
Unstable
Simpler controller (lower order)
Less complex solver method
More powerful hardware
Smaller eigenvalues
More complex solver method

Conclusion



— Model reduction can reduce the size of the eigenvalues.

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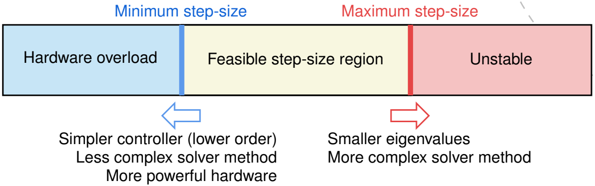

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- Increased solver order increases the stability region, but also the execution time.

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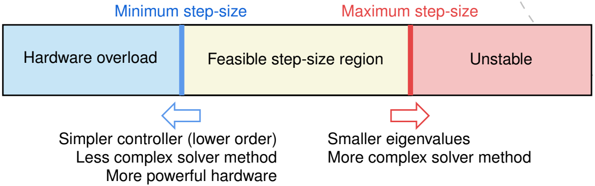

— Model reduction can reduce the size of the eigenvalues.
— Increased solver order increases the stability region, but also the execution time.
— Ultimately this is a tradeoff between moving an upper and lower limit for the step-size.

Conclusion

Suggested implementation procedure:

1. Design a controller and perform model reduction.
2. Find the maximum step-size providing stability for one or more chosen ERK methods.
3. An initial solver choice can be taken as the one which provides the best maximum step-size to computational complexity ratio. A rough estimate of the computational complexity can be found by simulations.
4. Run the controller at slightly below the maximum step-size, and reduce the step-size until the hardware is unable to perform the required calculations in time.
5. If no feasible step-size was found, one can reduce additional states or try another ERK method.

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That's all!
Any questions?

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