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ACC 2015
Chicago

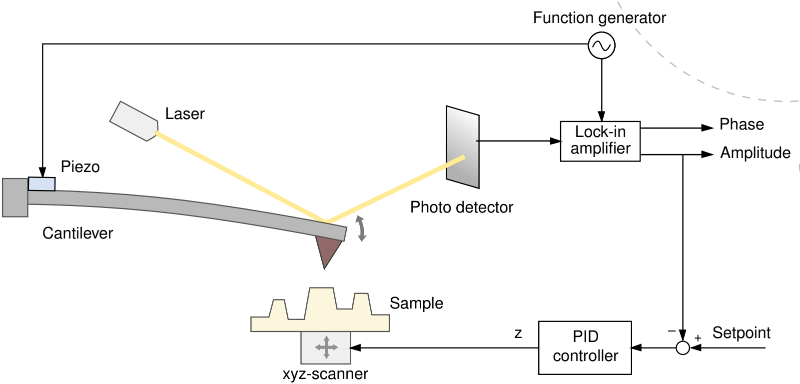
Topography and Force Estimation in Atomic Force Microscopy by State and Parameter Estimation

Michael R. P. Ragazzon, J. Tommy Gravdahl, Kristin Y. Pettersen, Arnfinn A. Eielsen

Department of Engineering Cybernetics,
Norwegian University of Science and Technology

ACC Chicago, July 1–3, 2015

Introduction – AFM AM Loop



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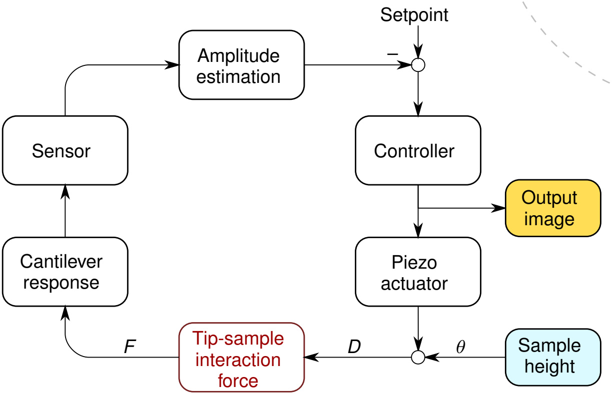

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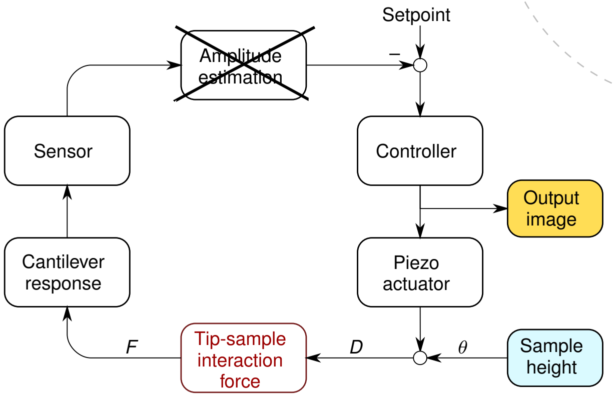

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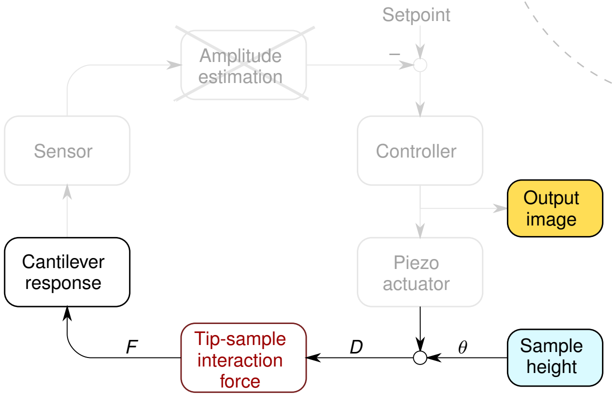

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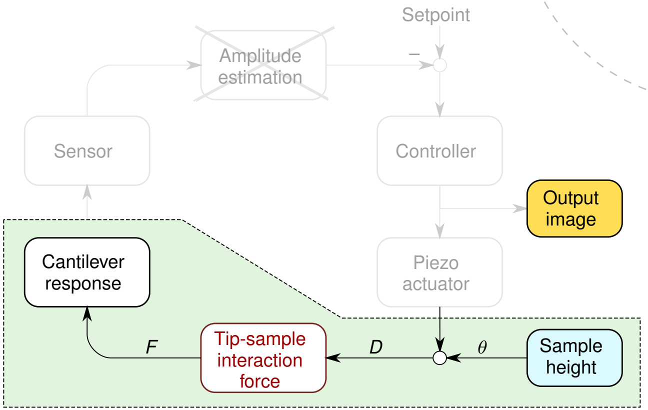

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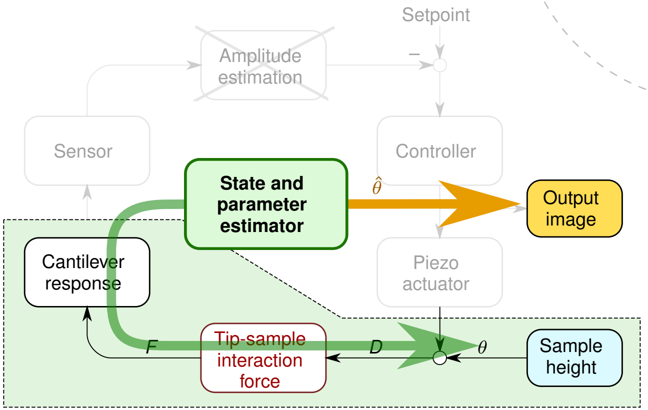

Introduction

— We will employ observers to directly estimate the topography
  - By considering the cantilever dynamics and interaction force

— Two observers were designed for the same purpose:
  - A nonlinear observer with well-defined exponential stability results.
  - An extended Kalman Filter for comparison.

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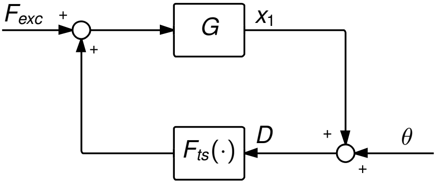
Outline

System Modeling

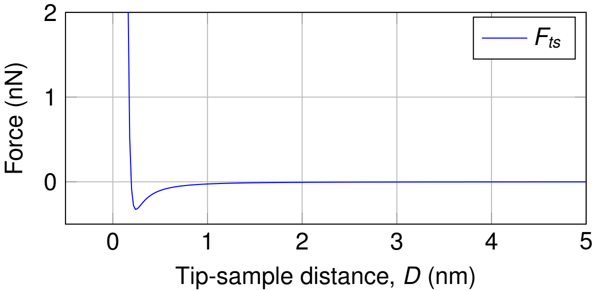


— G: Cantilever model from applied force, to tip deflection.
  - Second order harmonic oscillator.

— F_{ts}: Tip-sample interaction force: Modeled by Lennard-Jones potential.

System Modeling – Lennard-Jones potential

$$F_{ts}(D) = k_1 \left[\frac{\sigma^2}{D^2} - \frac{1}{30} \frac{\sigma^8}{D^8} \right] \tag{1}$$



System Modeling – State-space form

System can be expressed as an extended state-space model suitable for the state- and parameter estimator.

$$\begin{bmatrix} \dot{x} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} A & E \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \phi \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 1 \end{bmatrix} d \qquad (2)$$

$$A = \begin{bmatrix} 0 & 1 \\ -\omega_0^2 & -2\zeta\omega_0 \end{bmatrix}, \; B = E = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0 \end{bmatrix} \qquad (3)$$

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where

- the state $x \triangleq (x_1, x_2)^T$ represent the cantilever deflection and the deflection velocity
- the Lennard-Jones force F_{ts} has been introduced as a state ϕ
- d is the time-derivative of ϕ
- and the input u is the driving force of the cantilever.

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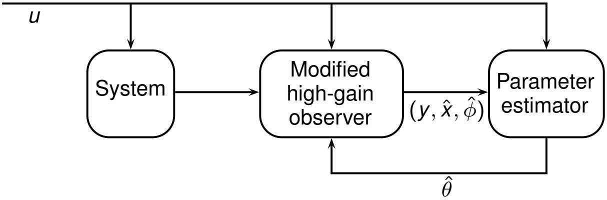
Nonlinear Observer

Based on Håvard F. Grip's article: *"Estimation of States and Parameters for linear systems with nonlinearly parameterized perturbations"* (2011).



Structure of the state- and parameter estimator. From (Grip et al., 2011).

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Nonlinear Observer – Modified High-Gain Observer

Observes the state $(\hat{x}, \hat{\phi})$ as if the parameter estimate $\hat{\theta}$ is perfectly known.

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + Bu + E\hat{\phi} + K_x(\varepsilon)(y - C\hat{x}) \\ \dot{z} &= -\frac{\partial g}{\partial \theta}\dot{\hat{\theta}} - \frac{\partial g}{\partial x}K_x(\varepsilon)(y - C\hat{x}) + K_\phi(\varepsilon)(y - C\hat{x}) \\ \hat{\phi} &= g(\hat{x}_1, \hat{\theta}) + z \end{aligned} \tag{4}$$

— Need to determine $K_x(\varepsilon), K_\phi(\varepsilon)$ such that the error dynamics of the observer are input-to-state stable.

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— Need to determine $K_x(\varepsilon), K_\phi(\varepsilon)$ such that the error dynamics of the observer are input-to-state stable.

Result

$$K_x(\varepsilon) = \begin{bmatrix} 4.0\epsilon^{-1} \\ 5.04\epsilon^{-2} \end{bmatrix} \tag{5}$$

$$K_\phi(\varepsilon) = 4.0\omega_0^2\epsilon^{-1} + 10.08\zeta\omega_0\epsilon^{-2} + 2.08\epsilon^{-3} \tag{6}$$

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Nonlinear Observer – Parameter Estimator

Need to find an update law

$$\dot{\hat{\theta}} = u_\theta(\nu, \hat{x}, \hat{\phi}, \hat{\theta}) \quad (7)$$

such that the origin of the error dynamics

$$\dot{\tilde{\theta}} = -u_\theta(\nu, \hat{x}, \hat{\phi}, \theta - \tilde{\theta}) \quad (8)$$

is uniformly exponentially stable whenever $\hat{x} = x$ and $\hat{\phi} = \phi$.

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Assumption 6

There exist a differentiable function $V_u : \mathbb{R}_{\geq 0} \times (\Theta - \Theta) \to \mathbb{R}_{\geq 0}$ *and positive constants* $a_1, \ldots, a_4$ *such that for all* $(t, \tilde{\theta}) \in \mathbb{R}_{\geq 0} \times (\Theta - \Theta)$,

$$a_1 \left\|\tilde{\theta}\right\|^2 \leq V_u(t, \tilde{\theta}) \leq a_2 \left\|\tilde{\theta}\right\|^2$$

$$\frac{\partial V_u}{\partial t}(t, \tilde{\theta}) - \frac{\partial V_u}{\partial \tilde{\theta}}(t, \tilde{\theta}) u_\theta(\nu, x, \phi, \theta - \tilde{\theta}) \leq -a_3 \left\|\tilde{\theta}\right\|$$

$$\left\|\frac{\partial V_u}{\partial \tilde{\theta}}(t, \tilde{\theta})\right\| \leq a_4 \left\|\tilde{\theta}\right\|$$

Furthermore, the update law (7) *ensures that if* $\hat{\theta}(0) \in \Theta$, *then for all* $t \geq 0, \hat{\theta} \in \Theta$.

Nonlinear Observer – Parameter Estimator

Need to find an update law

$$\dot{\hat{\theta}} = u_\theta(\nu, \hat{x}, \hat{\phi}, \hat{\theta}) \tag{7}$$

such that the origin of the error dynamics

$$\dot{\tilde{\theta}} = -u_\theta(\nu, \hat{x}, \hat{\phi}, \theta - \tilde{\theta}) \tag{8}$$

is uniformly exponentially stable whenever $\hat{x} = x$ and $\hat{\phi} = \phi$.

Assumption 6 satisifed by the following update law:

$$u_\theta(\nu, \hat{x}, \hat{\phi}, \hat{\theta}) = \text{Proj}\left(\Gamma M(\nu, \hat{x}, \hat{\theta})(\hat{\phi} - g(\nu, \hat{x}, \hat{\theta})\right) \tag{9}$$

$$M = \tfrac{1}{2} M_{\text{max}} \left[\tanh(M_{\text{rate}}(\hat{D} - D_M)) + 1\right] \tag{10}$$

Nonlinear Observer – Assumptions

— Bounded topography, well defined input signal and time-derivative

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— Operation in noncontact mode – allows the force profile to be represented by a monotonic function

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Nonlinear Observer – Assumptions

— Bounded topography, well defined input signal and time-derivative

— Operation in noncontact mode – allows the force profile to be represented by a monotonic function

— Known Lennard-Jones parameters

- Parameters appear linearly in Lennard-Jones function $\Rightarrow$ Method can easily be extended to simultaneously determine these.

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Nonlinear Observer – Stability Theorem

Theorem 1 (Abbreviated)

If all assumptions are satisfied and $\hat{\theta}(0) \in \Theta$, there exists $0 < \varepsilon^* \leq 1$ such that for all $0 < \varepsilon \leq \varepsilon^*$, the origin of the error dynamics of the observer and parameter estimator is exponentially stable.

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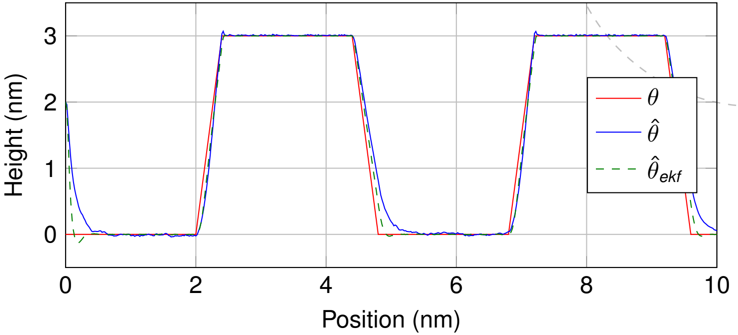
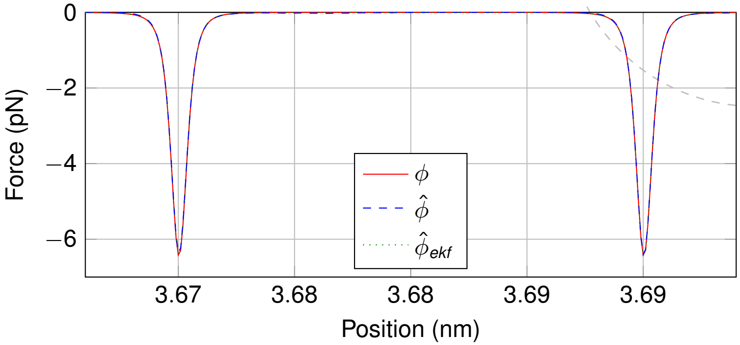
Simulation Results



Topography estimate with output noise.

Simulation Results



Estimated interaction force ϕ with noise.

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Conclusion

- New dynamic mode imaging method.
- Cantilever dynamics and interaction force "inverted" to estimate topography.
- Avoids bandwidth-limiting amplitude estimation.
- Exponentially stable observer.

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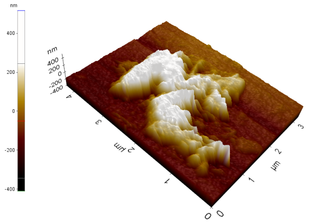
Further Work

— Experimental results.
— Include estimation of Lennard-Jones parameters.
— Use observer in feedback loop for control.
— Proper analysis of closed-loop bandwidth.



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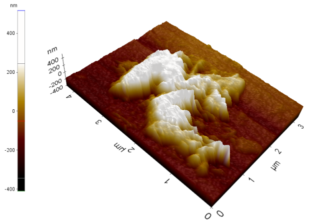
Questions?



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References

Håvard Fjær Grip, Ali Saberi, and Tor A Johansen. Estimation of states and parameters for linear systems with nonlinearly parameterized perturbations. *Systems & Control Letters*, 60(9):771–777, 2011.

Michael R P Ragazzon, J Tommy Gravdahl, Kristin Y Pettersen, and Arnfinn A Eielsen. Topography and Force Imaging in Atomic Force Microscopy by State and Parameter Estimation. In *American Control Conference, 2015*, 2015.

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